Calculation of the elastic properties of soft tissue through ultrasound elastography

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Aim

- Obtain a series of ultrasound images of the soft tissue under different compressions
- Calculate the elastic properties of the tissue as efficiently and accurately as possible.

My work

- Two different algorithms
  - A two stage algorithm
    (1) Image registration to give a displacement field.
    (2) Calculation of the elasticity parameters from a system of PDEs linking tissue displacement and elasticity parameters.
  - A one stage algorithm
    - Simultaneous displacement estimation and elasticity parameter recovery are carried out.

Two-stage algorithm

- Consider a material occupying $\Omega \in \mathbb{R}^3$
- Define spatial coordinate system $\mathbf{x} = (x_1, x_2, x_3) = (x, y, z)$
- Define $u(\mathbf{x}) = (u_1, u_2, u_3) = (u, v, w)$ to be the corresponding displacement field.
- Assume Linear elasticity, quasi-static deformation and plane-strain.
- Simplified problem in 2-d is

$$0 = \frac{\partial}{\partial x_i}\left(\lambda \frac{\partial u_k}{\partial x_k}\right) + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j}\right) + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_j}{\partial x_i}\right) \quad \text{for } i = 1, 2$$

- The Inverse Problem
  - Given $u(\mathbf{x})$, find $\mu(\mathbf{x})$ and $\lambda(\mathbf{x})$ to satisfy the governing equation.

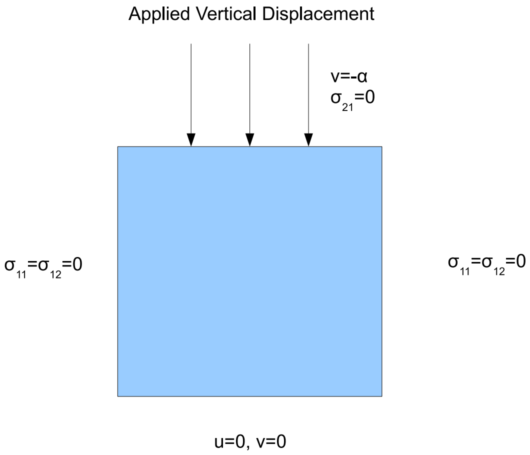
Two-stage algorithm

- Construct the discretised form on a finite element grid and apply boundary conditions to give the problem as

$$Au = f$$

where $A(\mu, \lambda)$, $f(\mu, \lambda)$ and u is a vector of both axial and lateral displacements

Boundary conditions



An iterative approach

- Assume Poisson's ratio $\nu = 0.49$
- Assume $\mu = exp(\gamma)$
- Problem is to minimise

$$F(\gamma) = \frac{1}{2} \sum_{i,j,k,l=1}^{N} (u_i(\gamma) - \hat{u}_i) W_{ij} M_{jk} W_{kl} (u_l(\gamma) - \hat{u}_l) + \alpha TV(\gamma)$$

- N =number of observation points in the deformed image.
- $u(\gamma)$ = theoretical displacements.
- $\hat{u}$ = observed displacements.
- W = weighting factor.
- M = symmetric positive definite matrix.
- α = regularisation parameter.
- $TV(\gamma)$ = a regularisation term based on the total variation of γ.

Elements of the algorithm

- Choice of regularisation term
- Choice of substitution of the elasticity parameter
- Choice of weighting factors
- An adjoint approach for constructing the gradient vector

An iterative approach

- Discretise the elasticity equation

$$\sum_j A_{ij}(\gamma)u_j = f_i$$

- As a minimisation problem, find γ that minimises

$$F(\gamma) = \frac{1}{2}\sum_{i,j,k,l=1}^{N}(u_i(\gamma) - \hat{u}_i)D_{ij}M_{jk}D_{kl}(u_l(\gamma) - \hat{u}_l) + \alpha(TV(\gamma))$$

- Many different approaches, but, in general, calculation of the gradient vector/Jacobian is required.
- Differentiating F leads to $g_I = \frac{\partial F}{\partial \gamma_I}$ and

$$\frac{\partial F}{\partial \gamma_I} = \sum_{i,j,k,l=1}^{N}(u_i(\gamma) - \hat{u}_i)D_{ij}M_{jk}D_{kl}\frac{\partial u_l}{\partial \gamma_I} + \alpha\frac{\partial}{\partial \gamma_I}(TV(\gamma)) \qquad \text{for } I = 1, \ldots, N$$

The adjoint method

- To use this, we need u_i and $\frac{\partial u_l}{\partial \gamma_I}$
- Through differentiation

$$A_{kl}\frac{\partial u_l}{\partial \gamma_I} = \frac{\partial f_k}{\partial \gamma_I} - \frac{\partial A_{kl}}{\partial \gamma I}u_l \quad \text{for } I = 1, \ldots, N$$

- Each loop requires $N \approx 10^4 - 10^5$ solves of the Forward Problem so reformulate as

$$g_I = \sum_{m,p} w_m \left(\frac{\partial f_m}{\partial \gamma_I} - \frac{\partial A_{mp}}{\partial \gamma_I}u_p \right) + \alpha \frac{\partial}{\partial \gamma_I}(TV(\gamma)) \quad \text{for } I = 1, \ldots, N$$

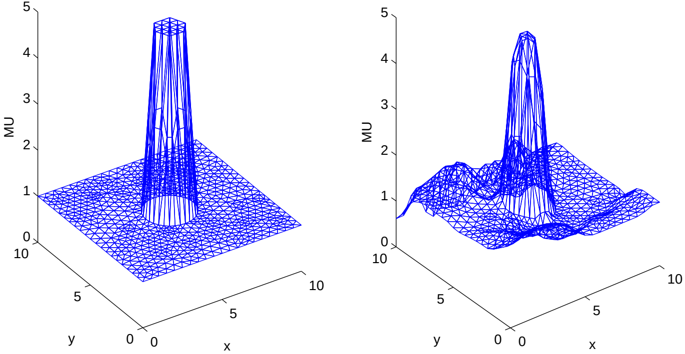
where

$$\sum_m A_{mk}w_m = -\sum_i (u_i - \hat{u}_i)M_{ij}D_{jk}$$

The adjoint method...

- Thus the steps in the algorithm are
  (1) Find u from initial elasticity equation.
  (2) Find w.
  (3) Use w, u to find g.
  (4) Use a gradient vector based optimisation algorithm to give the update to γ (L-BFGS implementation in MATLAB)
- These steps represent one iteration of the method.

Results: chosen MU, noise = 10%



One-stage algorithm

- This time we have the penalty function

$$F(\gamma) = \frac{1}{2}\sum_{i,j,k,l=1}^{N}(I(x_i)-I'(x_i'))W_{ij}M_{jk}W_{kl}(I(x_l)-I'(x_l'))+\alpha TV(\gamma)$$

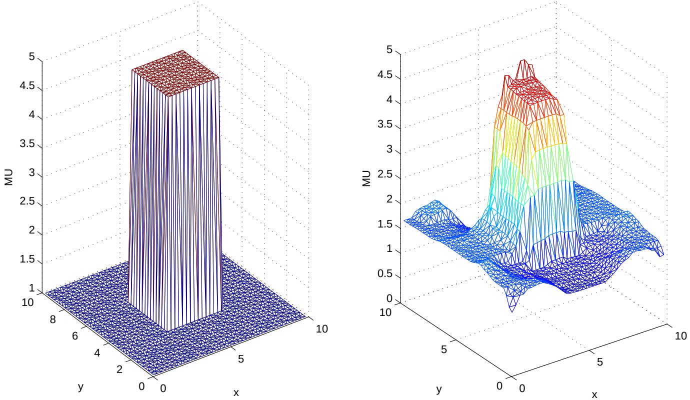
where

- $I(x_i)$ is the image intensity recorded from the initial image at point x_i.
- $I'(x_i')$ is the image intensity recorded in the compressed image at point x_i'.
- $x' = x + u(x)$.

One-stage algorithm

- Record $I(x_i)$ for all nodes in the non-deformed image.
- Initiate elasticity distributions of λ and μ.
- Solve the Forward problem to give the displacement field $u(x)$.
- Calculate $x_i' = x_i + u_i$.
- Compare the image intensity at x_i in the non-deformed image to the image intensity in the deformed image at x_i'.
- Calculate a suitable update of λ and μ
- Repeat until convergence.

Results: chosen MU, noise = 5%



Further work

- Perform comparisons on real data.
- 3-d extension

References

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