(A small sample of )
Solid mechanics in the Earth Sciences

Tarje Nissen-Meyer
UL Geophysics, Dept. Earth Sciences

Oxford Solid Mechanics Symposium
Oct 25 2013
**Why on Earth SM?**

<table>
<thead>
<tr>
<th>Continuum mechanics</th>
<th>Solid mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>The study of the physics of continuous materials with a defined rest shape.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fluid mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>The study of the physics of continuous materials which deform when subjected to a force.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describes materials that return to their rest shape after applied stresses are removed.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Plasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describes materials that permanently deform after a sufficient applied stress.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rheology</th>
</tr>
</thead>
<tbody>
<tr>
<td>The study of materials with both solid and fluid characteristics.</td>
</tr>
</tbody>
</table>

| Non-Newtonian fluids do not undergo strain rates proportional to the applied shear stress. |

| Newtonian fluids undergo strain rates proportional to the applied shear stress. |

There are four basic models that describe how a solid responds to an applied stress:

1. **Elastically** – When an applied stress is removed, the material returns to its undeformed state as described by the linear elasticity equations such as Hooke’s law.

2. **Viscoelastically** – These are materials that behave elastically, but also have damping effects and is converted in heat within the material resulting in a hysteresis loop in the stress-strain curve.

3. **Plastically** – Materials that behave elastically generally do so when the applied stress is lower than the yield stress, and does not return to its previous state. That is, deformation that occurs is permanent.

4. **Thermoelastically** – There is coupling of mechanical with thermal responses. In general, isothermal or adiabatic. The simplest theory involves the Fourier’s law of heat conduct.
Our lab
GOAL: understand Earth structure & dynamics

- Earth’s heat budget?
- Driving forces of plate tectonics?
- Existence & form of up-downwellings?
- Core dynamics and the magnetic field?
- Earthquake rupture?
- Brittle failure at large depths?
- Slow earthquakes

MODELS OF THE EARTH

SEISMIC IMAGING IS THE ESSENTIAL TOOL
People & research activities

R. Katz: dynamics of solid-fluid systems (e.g. magma)

B. Parsons: Geodesy (tectonics, earthquakes, volcanoes)

P. England: Tectonics, mechanical structure of subduction zones

T. Watts: marine crust and mantle structure, seismic refraction

S. Das: Earthquake rupture dynamics

J. Woodhouse: Global theoretical seismology

K. Sigloch: Seismic tomography and mantle dynamics

T. Nissen-Meyer: Computational seismology
Linear Elastodynamics (forward problem)

Hypothesised earthquake (source)

Seismograph (receiver)

Wave propagation using **AxiSEM**: Nissen-Meyer et al. 2013

**CPU-intensive and requires large storage**
Numerical solutions for seismic wave propagation

2D spectral–element method:
- discretization order: 4 to 6 (space); 2–4 (time)
- highly accurate: spectral convergence
- efficient: laptops, small clusters
- mesh honors discontinuities

(Nissen-Meyer et al., 2007, 2008)
Inverse problem: Linking waveforms to Earth structure

Wave source = assumed earthquake

Sensitivity kernels are CPU-intensive to compute and require large storage

Seismometer location

Nissen-Meyer 2013
Computational cost of inverse problems

- **20km**
  - Seismogram length: 10s
  - Sources: 50,000
  - Iterations: 30
  - RAM: 5 TB
  - CPU hours [million]: 345,000

- **8km**
  - CPU hours [million]: 27,000
  - RAM: 20 TB

- **6000km**
  - Seismogram length: 3000s
  - Sources: 1,000
  - Iterations: 20
  - RAM: 20 TB

- **20,000km**
  - CPU hours [million]: 27,000
  - RAM: 20 TB

**Exploration**

**Continental**

**Global**

**Basin**

Gordon Bell 2011
Gordon Bell 2007
Gordon Bell 1999
Gordon Bell 1988
Earthquake rupture dynamics

Modelling the Tohoku Earthquake (PhD P. Galvez, ETH)

✓ 3D spectral-element method SPECFEM3D
✓ Implemented rupture dynamics via split elements
✓ Rheology: slip-weakening friction laws
✓ Hexahedral meshing of multiple, branched non-planar faults
✓ Joint framework with elastodynamics and adjoints
✓ Extendable to other friction models (rate-and-state, plasticity)
Points of common interest

**Earthquake rupture:** Laboratory vs. modeling vs. observation

**Visco-elasticity:** Trade-off with elastic/thermal scattering

**Solid-fluid interactions:** partial melt, poro-elasticity

**Numerical methods:** GPU acceleration, high-order/local timestepping, meshing of complex media, Lattice-Boltzmann, Discontinuous Galerkin

**Multi-scale mechanics:** Homogenisation, multi-grid

**Optimisation:** Adjoint methods, uncertainties, data analysis